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The Evolution of Cooperation: Robustness to

Mistakes and Mutation

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Abstract: It is well known that repeated games present an embarassing multiplicity of equilibrium outcomes. Recently, Binmore and Samuelson (1992) have shown that cooperation is the unique evolutionary stable outcome when a game is played repeatedly by finite automata. This paper considers whether the evolution of cooperation is robust. Two types of perturbations are considered: mistakes by agents and mutation in strategies. Mistakes by agents are described assuming that the game is played by noisy automata. Mutation in strategies is accounted for by formalizing evolution as a 'modified' replicator dynamics which ensures that, at any point in time, every automaton is adopted by a positive number of players. Computer simulations indicate that, in the repeated prisoner's dilemma, cooperation is robust unless mistakes or mutation are very large.

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Introduction

It is well known that repeated games present an embarassing multiplicity of equilibrium outcomes [1]. This multiplicity is due to the variety of rules that can be adopted by the agents to play the game. One way of dealing with this multiplicity is to analyze the process of rules adoption trying to reduce the set of possible rules and thus the set of equilibrium outcomes. Here it is assumed that the adoption of rules is governed by evolution.

Evolutionary game theory is characterised by random matching of players whose actions are genetically dictated; changes in behaviour occur at the population level due to the higher reproductive rates of successful *phenotypes*. Utility is replaced by Darwinian fitness, measured by the number of *offsprings*, and a strategy is to be interpreted as a (genetically) programmed mode of behaviour [2].

When applying evolutionary game theory in economics one has to confront the fact that human behaviour is more sophisticated than animals' behaviour. Thus, while biologists assume that animals follow fixed strategy rules, economic agents are better described as finite automata whose behaviour is dictated by a set of procedures [3].

As it is explained at length in the next section, a *finite automaton* (Moore machine) is essentially a behavioral rule that tells the player how to behave in a repeated game [4]. The use of finite automata to describe players' behaviour allows to capture those aspects of bounded rationality that relates to the limits in the agents' processing capacities. In this framework, in fact, the complexity of a behavioral rule can be measured by the number of states of the corresponding automaton.

Recently, Binmore and Samuelson (1992) have shown that if players use

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[1]A version of the so-called "Folk Theorem" holds for infinitely repeated games with and without discounting and for finitely repeated games with incomplete information. The theorem shows that, any norm of behaviour which guarantees to the players a payoff greater then their security level can be sustained as an equilibrium if those who deviate from the norm can be punished (see Fudenberg-Maskin, 1986).

[2]See, for example, Friedman (1991).

[3]Alternatively, and equivalently, one can think of agents as using finite automata in their decisional processes. In what follows we refer to this second interpretation.

[4]The idea that players in a (repeated) game can be thought of as automata is due to Aumann (1981).

finite automata to play an infinitely repeated game, then cooperation is the unique evolutionary stable outcome. This paper studies the robustness of their result to mistakes by agents and to mutation in strategies.

Following Binmore-Samuelson, an automaton selection game is considered in which evolution selects the fittest automata [5]. They discuss evolutionary stability in terms of 'modified' evolutionary stable strategy. This notion of equilibrium modifies Maynard-Smith's (1982) definition of evolutionary stable strategy (ESS) to accomplish for lexicographic preferences of the 'metaplayers' (as Binmore and Samuelson call the players in the automaton selection game) and for the possibility of survival of mutants whose *observed* behaviour is the same as that of the population. Moreover, while ESS may not exist a 'modified' ESS always exists. They show that only cooperative automata can be part of a 'modified' ESS.

In this paper evolution is discussed within a selection-mutation model. At birth, individuals are randomly matched to play a repeated game. Each player uses a finite automaton of given complexity to play the game on his behalf. We allow for mistakes by agents assuming that automata are noisy. At each stage of the repeated game the automaton "suggests" to the player the action to be taken. With high probability this is consistent with the behavioural rule represented by the automaton. However, with a small probability the automaton makes a mistake and "suggests" an action at random. If the population of players is large and if the game is repeated long enough, then the randomness due to players' mistakes averages out.

Biologist formalize the genetic mechanism of natural selection using the replicator dynamics [6]. We modify this dynamics to account for a continuous flow of mutations which ensures that, at any point in time, every automaton is adopted by a positive number of players.

In the next section a model is presented that allows to discuss the evolution of behaviour in 2x2 symmetric (repeated) games played by two-state noisy automata. In this case, the payoff matrix of the automaton selection game is easily obtained by computing the payoffs for each automaton against each other. However, even in this simple case, the solution to the modified replicator dynamics is extremely difficult. Therefore, we have simulated it for the case in which the basic game is the prisoner's dilemma.

The results of the simulations show that cooperation is robust unless mistakes or mutations are very large. They also show that, if the probability of

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[5]The automaton selection game has been defined and first analyzed by Rubinstein (1986) and Abreu-Rubinstein (1988).

[6]See Hofbauer-Sigmund (1988).

mistakes is small, evolution selects for an automaton that cooperates if and only if both players took the same action in the previous period.

The rest of the paper is organised as follows: the model is presented in the next section; in section 2 the prisoner's dilemma is explicitly considered and in section 3 the results of the simulation are presented. Section 4 concludes.

1 The model

Consider a large population of individuals. At any given time a "generation" is alive. Each generation lives for N "years" (N large) and then dies. At birth individuals are randomly paired to play a 2x2 symmetric matrix game N times (once each year); so that a large number of games are played each year. It is assumed that players use finite automata to decide what to do in a given situation.

A *finite automaton* for player i consists of a finite set of states S^i, an initial state s^i_0, a reply function r^i and a transition function T^i. The reply function tells the player what action to take as a function of the current state: $a^i_t=r^i(s^i_t)$, $s^i_t \in S^i$. In what follows we assume that the reply function is such that $a^i_t=s^i_t$ $\forall i$. The transition function gives the state of the automaton at time t+1 as a function of the state at t and of the action taken by the other player: $s^i_{t+1}=T^i(s^i_t,a^j_t)$.

For any pair of automata $\{S^i,s^i_0,r^i,T^i\}$ and $\{S^j,s^j_0,r^j,T^j\}$ define:

(i) the state of the system at time t: $\mathbf{s}_t=(s^i_t,s^j_t)\in\mathbf{S}=S^i x S^j$;

(ii) a transition function $\mathbf{T}$ on $\mathbf{S}$ such that a transition from one state to the other according to $\mathbf{T}$ is possible only if it is possible for both T^i and T^j:

$$\mathbf{s}_{t+1}=\mathbf{T}\mathbf{s}_t$$

A fixed point (steady state) for the system is a state $\mathbf{s}^*$ such that $\mathbf{s}^*=\mathbf{T}\mathbf{s}^*$. Given $\mathbf{T}$ the steady state of the system is completely determined by the initial condition $\mathbf{s}_0$. This is so because the transition dynamics is deterministic.

If we allow for the possibility that sometimes automata make mistakes in the transition from one state to another [7] then the steady state of the system is independent from the initial conditions. Let p be the probability that an automaton makes a mistake, then the probability of going from s^i_t to $s^i_{t+1}=T^i(s^i_t,a^j_t)$ is $p(s^i_t,s^i_{t+1})=1-p+p/k$, where $k=card(S^i)$, and the probability of going from s^i_t to anyone of the other k-1 states in S^i is $p(s^i_t,s'^i_{t+1})=p/k$ for any

[7]Note that since $a^i_t=s^i_t$ $\forall i$, assuming that automata make mistakes in the transition phase is equivalent to assuming that they make mistakes when implementing an action.

$s'^i_{t+1} \in S^i$, $s'^i_{t+1} \neq s^i_{t+1}$ [8].

Analogously for the j-th automaton:

$$p(s^j_t, s^j_{t+1}) = 1 - p + p/k \quad s^j_{t+1} = T^j(s^j_t, a^j_t)$$
$$p(s^j_t, s'^j_{t+1}) = p/k \ \forall \ s'^j_{t+1} \in S^j, \ s'^j_{t+1} \neq s^j_{t+1}$$

If automata make mistakes, then the transition process $\mathbf{s}_{t+1} = \mathbf{T}_p \mathbf{s}_t$ is an ergodic Markov chain [9] and has a unique equilibrium distribution which is also the unique stationary distribution (Cox-Miller, 1965). In appendix 3 we show how to use the transition function $\mathbf{T}$ to calculate the average payoffs in the repeated prisoner's dilemma played by noisy automata of complexity two.

In what follows we assume that there is a fixed, exogenously given, upper bound on the complexity of the automata used by the players. Let $A^{(k)}$ be the set of all automata of complexity k and let n be the cardinality of this set.

Let $U(i_k, j_k)$ be the average "lifetime" payoff (i.e. the average payoff in N years) for an individual using automaton $i_k \in A^{(k)}$ when playing against an individual using automaton $j_k \in A^{(k)}$. Note that average lifetime payoffs depend on the number of repetition. However, as N approaches infinity the weight of a single year's payoff becomes negligible and $U(i_k, j_k)$ becomes arbitrarily close to the payoff of the infinitely repeated game, $U^\infty(i_k, j_k)$, computed as limit-of-the-means. In what follows $U(i_k, j_k)$ will be approximated by $U^\infty(i_k, j_k)$. Moreover, in the case of noisy automata, if the number of interactions between i_k and j_k, in any given generation, is sufficiently large, then the randomness due to players' mistakes averages out. It follows that in a large population in which generation's life is long enough, the matrix of average lifetime payoffs, $U^{(k)} = \{U(i_k, j_k)\}$, is the same in each generation and is approximately equal to $U^{\infty(k)}$.

The payoff matrix $U^{(k)}$ and the strategy space $A^{(k)}$ define an automaton selection game. It is assumed that the selection is operated by evolutionary forces. To find a dynamic equilibrium of the automaton selection game a state variable and a dynamic process must be defined. Let the state of the system be given by the distribution of automata in the current generation. Define

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[8] Note that we have assumed that the next state after a mistake is randomly determined according to a probability distribution that puts equal weight on each point in S^i.

[9] It is ergodic because there is a positive probability of going from each state to any other.

$x(t)=\{x_i(t), i=1..n, \Sigma_i x_i(t)=1\}$ to be this distribution in the t^h generation [10]; x(t) changes due to two forces: natural selection and mutations. We describe this intergenerational change using a *'modified' replicator dynamics*. According to this dynamics the majority of the population adjust their behaviour in response to current payoffs and move towards the most successful automata but a small proportion (h) of the population change their behaviour randomly.

Following the biological literature, we describe natural selection using the replicator dynamics according to which the growth rate of an automaton relative frequency is proportional to the difference between the average payoff for that automaton and the average payoff in the population. Formally the replicator dynamics can be expressed as (the dependence of x on time has been omitted to allow an easier reading):

$$(RD) \quad \dot{x}_i = [(Ux)_i - xUx]\, x_i$$

where $(Ux)_i = \Sigma_j U(i,j)x_j$.

Recurrent mutations, although very rare, continuously increase the entropy of the system; even if selection pressure at a given time acts against the survival of the i-th automaton, this will be maintained by mutations and thus kept available for changed circumstances. The role of this second term is to keep automata from extinction by rewarding automata with small shares and penalising those with high shares. We describe this second component by a mutation equation in which θ_{ij} is the mutation rate from the j-th automaton to the i-th with $\Sigma_i\theta_{ij}=1\ \forall j$. The probability of mutation from j to i in the whole population is $\theta_{ij}x_j$. Thus the growth rate of the i-th automaton due to mutations is

$$\dot{x}_i^{(M)} = \sum_{j=1}^{n} (\theta_{ij}x_j - \theta_{ji}x_i) \qquad (2)$$

Assume $\theta_{ii}=0$ and $\theta_{ij}=\theta_{kj}=\theta\ \forall i,j,k$, then $\theta=1/(n-1)$ (this follows from $\Sigma_i\theta_{ij}=1$). Substituting in the above expression gives [11]

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[10] In what follows we omit the indication about the complexity of the automata and use i and j instead of i_k and j_k.

[11] Since $\Sigma_j x_j=1$ it follows that $\Sigma_{j\neq i}x_j/(n-1) = (1-x_i)/(n-1)$

$$\dot{x}_i^{(M)} = \frac{1-x_i}{n-1} - x_i \quad \textbf{(3)}$$

The 'modified' replicator dynamics can thus be expressed by the following system of differential equations [12]:

$$(MRD) \quad \dot{x}_i = (1-h)[(Ux)_i - xUx]x_i + h\left(\frac{1-x_i}{n-1} - x_i\right) \quad i=1..n \quad 0 \le h \le 1$$

As h tends to 1 mutations increase their influence upon the system and when h=1 the only equilibrium of the system is $x_i = 1/n \ \forall i$.

A distribution x^* is a Nash equilibrium of the automaton selection game if $x^*Ux^* \ge yUx^* \ \forall y \in S_n$ [13]. In particular this is true for $y=e_i=(00..1..00)$ so that $x^*Ux^* \ge (Ux^*)_i$. A Nash equilibrium x^* is evolutionary stable (ESS) if for any other state $y \in S_n$:

(i) $x^*Ux^* > yUx^*$

or

(ii) $x^*Ux^* = yUx^*$ and $x^*Uy > yUy$

Nash equilibrium and ESS are static solution concepts.

A *dynamic equilibrium* for the 'modified' replicator dynamics is a fixed point of (MRD). A dynamic equilibrium x^* is (locally) *stable* if for any neighbourhood I of x^* there exists a neighbourhood I' such that any trajectory originated in I' remains in I, that is $\forall x(0) \in$ I' $x(t) \to x^* \in$ I. A dynamic equilibrium x^* is *asymptotically (locally) stable* if any trajectory that originates in I converges to it, that is if there exists an open neighbourhood $I(x^*)$ such that if $x(0) \in I$ then $x(t) \to x^*$. Note that asymptotical stability is a more stringent condition than stability.

In general the 'modified' replicator dynamics will have multiple equilibria. However, our simulations indicate that in some cases the system

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[12]Hofbauer-Sigmund (1988, p.250) consider a selection-mutation dynamics which is a particular case of MRD where the two components (selection and mutation) have the same weight.
With $m_{ij}=hu_{ij}$ and $\varepsilon_{ij}=h\theta_{ij}$ MRD and Hofbauer-Sigmund's equation (25.4) are formaly the same. However, we have $\Sigma_i\varepsilon_{ij}=h$ and not 1 as in their model. This makes clear that we are considering a case in which mutations are rare and therefore their impact on evolution is that of a perturbation on a selection process.

[13]S_n is the n-dimensional simplex.

may be globally stable. Looking at the simulations' results (see section 3) it seems that global stability is easier to obtain when h>0 and p is small. Local stability has been observed in all simulations but one; as it is shown in figure 1, in fact, the 'modified' replicator dynamics need not converge but can exhibit a cyclical behaviour. These results suggest that the 'modified' replicator dynamics can exhibit global convergence, local stability or cyclical behaviour depending on the initial conditions and on the value of h.

Some properties of the modified replicator dynamics are given in appendix 1.

2. The prisoner's dilemma

In this section we apply the model of the previous section to investigate the evolution of behaviour in the repeated prisoner's dilemma (see figure below, where C means Cooperate and D defect) played by two-state automata.

	C	D
C	2 2	-1 3
D	3 -1	0 0

The Prisoner's Dilemma

A complete list of two-state automata able to play the prisoner's dilemma is given in appendix 2 [14].

The first automaton is the one that "always defect". Number 9 represent the GRIM-strategy: "start by cooperating and play cooperatively unless the other deviates, in this case defect in all subsequent periods". The 10th automaton says: "do what the other did in the previous period", this is TIT-FOR-TAT. Automaton number 11 is TAT-FOR-TIT which suggests: "repeat

[14] There are 16 (32 considering the initial condition) rules of complexity two able to play a 2x2x game. Binmore-Samuelson (1992) consider only 26 automata because, without mistakes, the observed behaviour of the three automata that start cooperating and reply cooperatively to any choice of the opponent (automata 13, 14, 15) is indistinguishable from the behaviour of the automaton that always cooperate (automaton 16). Also the observed behaviour of the three automata that start defecting and defect after any choice of the opponent (automata 21, 25, 29) is indistinguishable from the behaviour of the automaton that always defect (automaton 17).

your previous action if the outcome was satisfactory otherwise change action". "Always cooperate" is the last automaton, the 16th.

Following the procedure illustarted in appendix 3 we have computed the payoff matrix $U^{(2)}$ for different probabilities of mistake. These are shown in tables 1 to 6 for p=0, .01, .05, .1, .2, .4 respectively [15]. The matrices in tables 2 to 5 are 16x16 because two noisy automata with different initial state but otherwise identical are payoff-equivalent. Thus when automata make mistakes the generic element U_{ij} of the payoff matrix $U^{(2)}$ is:

$$U_{i,j}=U_{i,j+16}=U_{i+16,j}=U_{i+16,j+16} \quad i,j=1..16$$

that is, the 32x32 matrix is formed by four 16x16 identical submatrices and only one of them need to be considered in the automaton selection game. The steps needed to compute the payoff matrix $U^{(2)}$ for the prisoner's dilemma are illustrated in appendix 3.

3. The results

We have simulated the 'modified' replicator dynamics for the automaton selection games of tables 1-6 for a variety of initial conditions. Tables 7 to 12 summarise the results. The first column in each table gives the distribution of automata in the population at time t=0. V_i means that the initial distribution is $x_i(0)=1-0.01(n-1)$ and $x_j(0)=0.01$ $\forall j \neq i$. E means that the initial distribution is $x_i(0)=1/n$ $\forall i$. The second column gives the steady state of the 'modified' replicator dynamics with h=.01 obtained from each initial distribution in the first column. The third column gives the corresponding steady state for the replicator dynamics, that is for the model without mutations.

When automata do not make mistakes the 'modified' replicator dynamics with h=.01 converges to $x_9=.54$, $x_{10}=.10$, $x_{11}=.12$, $x_{12}=.08$, $x_{13}=x_{14}=x_{15}=x_{16}=.04$ for all the initial distributions considered.

A comparison of the results obtained for h=.01 with those obtained for h=0 (see table 7) suggests that the role of mutations is to help in reaching a unique stable equilibrium. Note that table 7 is consistent with Binmore-Samuelson's result according to which in two-player infinitely repeated games evolution leads to the survival of cooperative behaviour. In fact, all the automata that survive natural selection in the game of table 1 get, in equilibrium, a payoff of 2.

Table 8 summarises the convergence results obtained when p=.01, that is for the game of table 2. In this game there are three Nash equilibria in pure strategies, $x_1=1$, $x_9=1$ and $x_{11}=1$, but only the last one is an ESS. If there are no mutations (h=0) then the system converges to the ESS ($x_{11}=1$) or to a Nash

[15]The payoffs in tables 1-6 are approximated to the first decimal place, however in the simulations we have used their true values.

equilibrium (x_9=1) depending on the initial distribution. Adding mutations (h=.01) the 'modified' replicator dynamics converges to a dynamic equilibrium "close" to the ESS for all the initial distributions considered.

Looking at the payoff matrix in table 2 one can see that the 1-st and the 9-th automata are best replies against the 3-rd and the 5-th; therefore starting from V3 or V5 both x_1 and x_9 grow. Since the 11-th automaton gets a negative payoff against the 1-st, in the system without mutations x_{11} quickly becomes extinct. Figure 2 and 2b show that starting from V5 and without mutations the 11-th automaton is quickly eliminated; both x_1 and x_9 increase until x_1=x_9=.5 and then, while x_9 slowly grows, x_1 decreases [16]. On the contrary, if h>0 then the 11-th automaton is never eliminated and once the environment becomes more favourable (which is when x_i i≥12 start growing) x_{11} increases (see figure 3).

Table 9 summarises the convergence results obtained when p=.05, that is for the game of table 3. In this game there are three Nash equilibria in pure strategies, x_1=1, x_9=1 and x_{11}=1; which are also ESSs [17]. The results for this game are very similar to those in table 8. The replicator dynamics always converges to an ESS; to which one depends on the initial conditions. The 'modified' replicator dynamics with h=.01 converges to a dynamic equilibrium (x_{11}=.98) "close" to the ESS with higher payoff for all the initial distributions considered.

Figure 4 and 5 show the evolution of the system starting from V1 for h=.01 and h=0 respectively. In figure 4 mutations, by preventing automata from becoming extinct, allow the 10-th automaton to grow thus creating a favourable environment (high proportions of the i-th automaton, i≥12) in which the 11-th automaton can prosper. This is unlike the system without mutations in which x_1=1 is locally asymptotically stable (see figure 5).

Table 10 summarises the convergence results obtained when p=.1, that is for the game of table 4. In this game there are three ESSs in pure strategies, x_1=1, x_9=1 and x_{11}=1. The replicator dynamics always converges to an ESS; to which one depends on the initial conditions. Adding mutations (h=.01) the system converges to a dynamic equilibrium (x_{11}=.98) "close" to the best ESS for all the initial distributions considered except V6 and V9; starting from V6 and V9 the system converges to x_9=.85, x_{11}=.04. However, if h=.001 and the system starts from V6 or V9 it converges to x_{11}=1.

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[16]This is because U(1,1)=U(1,9)=U(9,1)=.01 and U(9,9)=.015.

[17]For this and the following games, to say if a Nash equilibrium is an ESS we looked at the payoffs with more decimal places than those reported in the tables.

Table 11 summarises the convergence results obtained when p=.2, that is for the game of table 5. Also in this game x_1=1, x_9=1 and x_{11}=1 are ESSs. The replicator dynamics converges to the best ESS if the system starts nearby (and also from E) and to x_1=1 or x_9=1 otherwise. In this game the dynamic equilibrium of the 'modified' replicator dynamics with h=.01 is not "close" to the best ESS. In fact, starting from all the initial distributions considered, except for V11, the steady state of the dynamics with h=.01 is x_9=.88, x_{10}=.03, x_1=.01, x_i=.01 i=11..16 (starting from V11 the steady state is x_{11}=.96).

Table 12 summarises the convergence results obtained when p=.4, that is for the game of table 6. In this game there are only two Nash equilibria in pure strategies (x_1=1 and x_9=1), both of which are ESSs. The replicator dynamics converges to x_1=1 for all the initial conditions considered, except V6 and V9, while starting from V6 or V9 the steady state reached is x_9=1. The 'modified' replicator dynamics with h=.01 converges to x_1=.94 starting from all the initial distributions considered, except V6, V9 and V10, and to x_9=.90 starting from V6, V9 or V10. In this game the steady state reached by the 'modified' replicator dynamics and by the replicator dynamics are "close" for all initial distributions except V10. This may be due to the fact that in a highly random environment mutations do not matter much.

4. Conclusions

From the simulation results illustrated in the previous section we can conclude that:

(i) In the unperturbed system (no mistakes nor mutations) the only survivors are GRIM and TIT-FOR-TAT [18]. This is consistent with Binmore-Samuelson's result. However, the dynamic equilibrium of the unperturbed system is not unique and it depends on the initial distribution of automata.

(ii) Introducing mutations into the selection dynamics (but mantaining the assumption of no mistakes) reduces this multiplicity and a unique equilibrium is selected in which a high share of the population is of the GRIM-type (with some TIT-FOR-TAT and some TAT-FOR-TIT).

(iii) In the model with noisy automata cooperation survives unless the probability of mistakes is very high. For p≤.1 the population is almost entirely composed of TAT-FOR-TIT; when p=.2 GRIM is the fittest; for p=.4 ALWAYS-DEFECT is the unique survivior. This is consistent with Fudenberg and Maskin (1990) who show that in 2x2 symmetric repeated games played by noisy machines, if mistakes are rare then evolutionary stability implies cooperation. If the probability of mistakes is too high then their assumption of

[18] "Start playing cooperatively and then do what the other did in the previous period".

lexicographic preferences is not satisfied and the efficient outcome is not guaranteed.

Compared with Axelrod's (1984) tournament, our simulations indicate that in a noisy environemnt TIT-FOR-TAT never survives. The reason for the non-robustness of TIT-FOR-TAT is that occasional mistakes between two TIT-FOR-TAT automata trigger a sequence of mutual punishment. On the contrary, an occasional deviation from cooperation between two TAT-FOR-TIT automata causes a round of mutual defection followed by a return to cooperation.

If mistakes are interpreted as experimentation undertaken to *identify* the opponent, then the success of TAT-FOR-TIT can be seen as the result of its ability to use a costless signal (a 'secret handshake' in Robson (1990) terminology) to recognise and to be recognised by other automata of the same type [19].

[19]In the case of TAT-FOR-TIT the signal would be T(DD)=C, that is after a mutual deviation switch to cooperation.

Appendix 1

Here are some properties of the 'modified' replicator dynamics:

P1: *All dynamic equilibria of the 'modified' replicator dynamics are in the interior of the simplex.*

proof: suppose not and suppose that in a dynamic equilibrium $x^*_i=0$ for some i; then

$$\dot{x}_i^*=\frac{h}{N-1}>0$$

contradicting the hypothesis that x^* is a dynamic equilibrium.

P2: *In general, a dynamic equilibrium of the 'modified' replicator dynamics is not a Nash equilibrium.*

proof: if x^* is a dynamic equilibrium of the 'modified' replicator dynamics then

$$(1)\quad (1-h)x_i^*[(Ux^*)_i-x^*Ux^*]=h\frac{1-Nx_i^*}{N-1}\quad \forall i$$

that is

$$(2)\quad (Ux^*)_i-x^*Ux^*=\frac{h}{1-h}\frac{1-Nx_i^*}{x_i^*(N-1)}\quad \forall i$$

Suppose that x^* is a Nash equilibrium then, by definition, $x^*Ux^*\geq(Ux^*)_i\ \forall i$. This implies that $x^*_i\leq 1/N\ \forall i$ which contradicts $\Sigma_i x_i=1$.

P3: *Let x^* be a dynamic equilibrium of the 'modified' replicator dynamics, then*

$$(Ux^*)_i>x^*Ux^*\quad if\quad x_i^*>\frac{1}{N}$$

and

$$(Ux^*)_i<x^*Ux^*\quad if\quad x_i^*<\frac{1}{N}$$

proof: the proposition follows from equation (2) of P2.

From the same equation also follows that

As h-->0 the dynamic equilibrium of the 'modified' replicator dynamics tends

$$0<(Ux)_i<xUx+\frac{h}{1-h} \quad \forall i$$

to the Nash equilibrium.

For the replicator dynamics the following can be proven (see van Damme, 1991):

P4: *If x is a symmetric Nash equilibrium of U then x is a dynamic equilibrium, but the converse need not be true.*

P5: *If x is a stable dynamic equilibrium then x is a symmetric Nash equilibrium of U, but the converse need not be true.*

P6: *Every ESS is an asymptotically stable equilibrium but not conversely.*

Appendix 2

The entry ij in the following matrix is the state of the i-th automaton at t+1 if its state at t and the opponent action at t are as in column j (see section 1). Remember that automata from 17 to 32 are equivalent to, respectively automata 1 to 16 except for the initial state (the first 16 automata start with C the last with D).

	CC	CD	DC	DD
1	D	D	D	D
2	D	D	C	D
3	D	D	D	C
4	D	D	C	C
5	D	C	D	D
6	D	C	C	D
7	D	C	D	C
8	D	C	C	C
9	C	D	D	D
10	C	D	C	D
11	C	D	D	C
12	C	D	C	C
13	C	C	D	D
14	C	C	C	D
15	C	C	D	C
16	C	C	C	C

Appendix 3

For any pair of two-state automata able to play the prisoner's dilemma there are four possible states of the system: s_1=CC, s_2=CD, s_3=DC, s_4=DD, where $\Gamma\Delta$ (Γ,Δ=C,D) indicates that the current state (action) of the automaton used by the first player is Γ and the current state (action) of the automaton used by the second player is Δ.

Let $x_\alpha=\text{prob}(s_\alpha)$, $\alpha=1..4$ and $x=(x_1,x_2,x_3,x_4)$ and let $q_{\alpha\beta}=\text{prob}(s_\alpha \dashrightarrow s_\beta)$ with $\Sigma_\beta q_{\alpha\beta}=1$ $\alpha=1..4$ (starting from any state the system has to go somewhere); $q_{\alpha\beta}$ depends on the automata used by the players as well as on the probability that automata make mistakes: $q_{\alpha\beta}=q_{\alpha\beta}(i_2,j_2,p)$. Consider the following system of linear equations:

$$Mx=b$$

where $b=(0,0,0,0,1)$ and M is given by:

$q_{11}-1$	q_{21}	q_{31}	q_{41}
q_{12}	$q_{22}-1$	q_{31}	q_{41}
q_{13}	q_{23}	$q_{33}-1$	q_{43}
q_{14}	q_{24}	q_{34}	$q_{44}-1$
1	1	1	1

The last equation ($\Sigma_\alpha x_\alpha=1$) indicates that, at any time the system must be in one of the four states) [20]. The solution of the system, x^*, is the unique stationary probability distribution over the states.

The payoff for a player using automaton i_2 against j_2 in the prisoner's dilemma played by automata that make mistakes with probability p is:

$$U(i_2,j_2,p)=2x^*_1 - x^*_2 + 3x^*_3 \ [21]$$

where $x^*_\alpha=x^*_\alpha(i_2,j_2,p)$

As an example we solve the system for the case in which player 1 uses the 9-th automaton and player 2 uses the 10-th automaton (see appendix 1). If p is the probability that an automaton makes a mistake then the probability of

[20]This condition implies that in Mx=b one equation is linearly dependent from the others and can therefore be eliminated leaving us with a system of four equations in four unknowns (the x_i).

[21]Since the game is symmetric the payoff for a player adopting automaton j_2 against i_2 is:

$$U(j_2,i_2,p)=2x_1 + 3x_2 - x_3$$

a correct action is q_c=(1-p)+p/2 and that of an incorrect action is p_c=1-q_c [22].

To calculate $q_{\alpha\beta}=q_{\alpha\beta}(i_2,j_2,p)$ α,β=1..4 we can use the diagrams in figures 6a to 6d. In figures 6a and 6b the transition functions respectively for the 9-th and 10-th automaton are shown graphically. In figure 6c the transition function **T** obtained from T^9 and T^{10} is shown. This is obtained by keeping only the arrows that are common to both T^9 and T^{10}. **T** describes the evolution of the system without mistakes. As it can be seen from figure 6c, if the system starts in CC then it stays there forever; but if it starts from any other state then it converges to DD. Figure 6d shows the transition function $\mathbf{T_p}$ corresponding to **T** in figure 6c. If automata make mistakes then all the transition from any state to any other occur with positive probability. The bold-line arrows correspond to the arrows in figure 6c i.e. to the case in which neither automata make mistakes and they represent a probability $q_{\alpha\beta}=q_c^2$. The solid-line arrows correspond to the case in which only one automaton makes a mistake and represent a probability $q_{\alpha\beta}=q_c p_c$. The dotted-line arrows indicate transitions that occur when both automata make a mistake; they represent a probability $q_{\alpha\beta}=p_c^2$. It follows that the matrix M is given by:

q_c^2-1	$p_c q_c$	p_{c2}	p_{c2}
$p_c q_c$	p_{c2}-1	$p_c q_c$	$p_c q_c$
$p_c q_c$	q_c^2	$p_c q_c$-1	$p_c q_c$
p_{c2}	$p_c q_c$	q_c^2	q_c^2-1
1	1	1	1

Solving the system Mx=b gives:

$$x_1^*=x_2^*=\frac{p_c}{2p_c+1} \qquad x_3^*=\frac{2q_c p_c}{2p_c+1} \qquad x_4^*=\frac{q_c^2+p_c^2}{2p_c+1}$$

[22]This is because we have assumed that when making a mistake the action taken is randomly drawn from a probability distribution that puts equal weight on each action, there is a probability of p/2 that after a mistake the correct action is taken.

For example, if p=.1, which implies q_c=.95 and p_c=.05, then

$$x_1^* = x_2^* = \frac{5}{110} \quad x_3^* = \frac{9.5}{110} \quad x_4^* = \frac{90.5}{110}$$

and the payoffs are

$$U(9_2,10_2)=.3054 \qquad U(10_2,9_2)=.1409$$

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Table 1 probability of mistake p = 0

0.0	0.0	1.5	1.5	3.0	3.0	3.0	3.0	0.0	0.0	1.5	1.5	3.0	3.0	3.0	3.0
0.0	1.0	0.7	1.0	0.0	0.0	1.7	1.7	0.0	1.0	0.7	1.0	2.5	2.5	2.5	2.5
-0.5	0.7	1.0	1.0	3.0	3.0	3.0	3.0	-0.5	0.7	1.7	1.7	3.0	3.0	3.0	3.0
-0.5	1.0	1.0	1.0	-0.5	1.0	1.0	1.0	-0.5	1.0	1.0	1.0	2.5	2.5	2.5	2.5
0.0	0.0	1.5	1.5	3.0	3.0	3.0	3.0	0.0	0.0	1.5	1.5	3.0	3.0	3.0	3.0
-1.0	0.0	-1.0	1.0	0.0	0.0	1.7	1.7	-1.0	1.3	-1.0	1.3	2.5	2.5	2.5	2.5
-1.0	0.3	1.0	1.0	3.0	3.0	3.0	3.0	-1.0	1.0	1.7	1.7	3.0	3.0	3.0	3.0
-1.0	0.3	-1.0	1.0	-1.0	0.3	1.0	1.0	-1.0	1.3	-1.0	1.3	2.5	2.5	2.5	2.5
0.0	0.0	1.5	1.5	3.0	3.0	3.0	3.0	0.0	0.0	1.5	1.5	3.0	3.0	3.0	3.0
0.0	1.0	0.7	1.0	0.0	1.3	1.0	1.3	0.0	1.0	0.7	1.0	2.0	2.0	2.0	2.0
-0.5	0.7	0.3	1.0	3.0	3.0	3.0	3.0	-0.5	0.7	2.0	2.0	3.0	3.0	3.0	3.0
-0.5	1.0	0.3	1.0	-0.5	1.3	0.3	1.3	-0.5	1.0	2.0	1.0	2.0	2.0	2.0	2.0
0.0	0.0	1.5	1.5	3.0	3.0	3.0	3.0	0.0	0.0	1.5	1.5	3.0	3.0	3.0	3.0
-1.0	0.5	-1.0	0.5	-1.0	0.5	-1.0	0.5	-1.0	2.0	-1.0	2.0	2.0	2.0	2.0	2.0
-1.0	0.5	-1.0	0.5	3.0	3.0	3.0	3.0	-1.0	2.0	2.0	2.0	3.0	3.0	3.0	3.0
-1.0	0.5	-1.0	0.5	-1.0	0.5	-1.0	0.5	-1.0	2.0	-1.0	2.0	2.0	2.0	2.0	2.0

0.0	0.0	1.5	1.5	0.0	0.0	3.0	3.0	0.0	0.0	1.5	1.5	3.0	3.0	3.0	3.0
0.0	0.0	0.7	1.0	0.0	0.0	1.7	1.7	0.0	1.0	0.7	1.0	2.5	2.5	2.5	2.5
-0.5	0.7	1.0	1.0	-0.5	3.0	1.0	1.0	-0.5	0.7	1.7	1.7	3.0	3.0	3.0	3.0
-0.5	1.0	1.0	1.0	-0.5	1.0	1.0	1.0	-0.5	1.0	1.0	1.0	2.5	2.5	2.5	2.5
0.0	0.0	1.5	1.5	0.0	0.0	3.0	3.0	0.0	0.0	1.5	1.5	3.0	3.0	3.0	3.0
0.0	0.0	-1.0	1.0	0.0	0.0	1.7	1.7	-1.0	1.3	-1.0	1.3	2.5	2.5	2.5	2.5
-1.0	0.3	1.0	1.0	-1.0	0.3	1.0	1.0	-1.0	1.0	1.7	1.7	3.0	3.0	3.0	3.0
-1.0	0.3	1.0	1.0	-1.0	0.3	1.0	1.0	-1.0	1.3	-1.0	1.3	2.5	2.5	2.5	2.5
0.0	0.0	1.5	1.5	0.0	3.0	3.0	3.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
0.0	1.0	0.7	1.0	0.0	1.3	1.0	1.3	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
-0.5	0.7	0.3	1.0	-0.5	3.0	0.3	3.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
-0.5	1.0	0.3	1.0	-0.5	1.3	0.3	1.3	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
-1.0	0.5	-1.0	0.5	-1.0	0.5	-1.0	0.5	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
-1.0	0.5	-1.0	0.5	-1.0	0.5	-1.0	0.5	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
-1.0	0.5	-1.0	0.5	-1.0	0.5	-1.0	0.5	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
-1.0	0.5	-1.0	0.5	-1.0	0.5	-1.0	0.5	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0

0.0	0.0	1.5	1.5	0.0	0.0	3.0	3.0	0.0	0.0	1.5	1.5	0.0	0.0	3.0	3.0
0.0	0.0	0.7	1.0	0.0	0.0	1.7	1.7	0.0	0.0	0.7	1.0	0.0	0.0	2.5	2.5
-0.5	0.7	1.0	1.0	-0.5	3.0	1.0	1.0	-0.5	0.7	1.7	1.7	-0.5	3.0	3.0	3.0
-0.5	1.0	1.0	1.0	-0.5	1.0	1.0	1.0	-0.5	1.0	1.0	1.0	-0.5	2.5	2.5	2.5
0.0	0.0	1.5	1.5	0.0	0.0	3.0	3.0	0.0	0.0	1.5	1.5	0.0	0.0	3.0	3.0
0.0	0.0	-1.0	1.0	0.0	0.0	1.7	1.7	0.0	0.0	-1.0	1.3	0.0	0.0	2.5	2.5
-1.0	0.3	1.0	1.0	-1.0	0.3	1.0	1.0	-1.0	1.0	1.7	1.7	-1.0	3.0	3.0	3.0
-1.0	0.3	1.0	1.0	-1.0	0.3	1.0	1.0	-1.0	1.3	-1.0	1.3	-1.0	2.5	2.5	2.5
0.0	0.0	1.5	1.5	0.0	0.0	3.0	3.0	0.0	0.0	1.5	1.5	0.0	0.0	3.0	3.0
0.0	0.0	0.7	1.0	0.0	0.0	1.0	1.3	0.0	0.0	0.7	1.0	0.0	0.0	2.0	2.0
-0.5	0.7	0.3	1.0	-0.5	3.0	0.3	3.0	-0.5	0.7	2.0	2.0	-0.5	3.0	2.0	2.0
-0.5	1.0	0.3	1.0	-0.5	1.3	0.3	1.3	-0.5	1.0	2.0	2.0	-0.5	2.0	2.0	2.0
0.0	0.0	1.5	1.5	0.0	0.0	3.0	3.0	0.0	0.0	1.5	1.5	0.0	0.0	3.0	3.0
0.0	0.0	-1.0	0.5	0.0	0.0	-1.0	0.5	0.0	0.0	-1.0	2.0	0.0	0.0	2.0	2.0
-1.0	0.5	-1.0	0.5	-1.0	0.5	-1.0	0.5	-1.0	2.0	2.0	2.0	-1.0	2.0	2.0	2.0
-1.0	0.5	-1.0	0.5	-1.0	0.5	-1.0	0.5	-1.0	2.0	2.0	2.0	-1.0	2.0	2.0	2.0

0.0	0.0	1.5	1.5	0.0	3.0	3.0	3.0	0.0	0.0	1.5	1.5	0.0	3.0	3.0	3.0
0.0	1.0	0.7	1.0	0.0	0.0	1.7	1.7	0.0	1.0	0.7	1.0	0.0	2.5	2.5	2.5
-0.5	0.7	1.0	1.0	-0.5	3.0	1.0	3.0	-0.5	0.7	1.7	1.7	-0.5	3.0	3.0	3.0
-0.5	1.0	1.0	1.0	-0.5	1.0	1.0	1.0	-0.5	1.0	1.0	1.0	-0.5	2.5	2.5	2.5
-1.0	0.0	-1.0	1.5	-1.0	0.0	-1.0	3.0	-1.0	0.0	-1.0	1.5	-1.0	3.0	-1.0	3.0
-1.0	0.0	-1.0	1.0	-1.0	0.0	-1.0	1.7	-1.0	1.3	-1.0	1.3	-1.0	2.5	-1.0	2.5
-1.0	0.3	-1.0	1.0	-1.0	0.3	-1.0	1.0	-1.0	1.0	-1.0	1.7	-1.0	3.0	-1.0	3.0
-1.0	0.3	-1.0	1.0	-1.0	0.3	-1.0	1.0	-1.0	1.3	-1.0	1.3	-1.0	2.5	-1.0	2.5
0.0	0.0	1.5	1.5	0.0	3.0	3.0	3.0	0.0	0.0	1.5	1.5	0.0	3.0	3.0	3.0
0.0	1.0	0.7	1.0	0.0	1.3	1.0	1.3	0.0	1.0	0.7	1.0	0.0	2.0	2.0	2.0
-0.5	0.7	0.3	1.0	-0.5	3.0	0.3	3.0	-0.5	0.7	2.0	2.0	-0.5	3.0	2.0	3.0
-0.5	1.0	0.3	1.0	-0.5	1.3	0.3	1.3	-0.5	1.0	2.0	1.0	-0.5	2.0	2.0	2.0
-1.0	0.5	-1.0	0.5	-1.0	0.5	-1.0	0.5	-1.0	2.0	-1.0	2.0	-1.0	2.0	-1.0	2.0
-1.0	0.5	-1.0	0.5	-1.0	0.5	-1.0	0.5	-1.0	2.0	-1.0	2.0	-1.0	2.0	-1.0	2.0
-1.0	0.5	-1.0	0.5	-1.0	0.5	-1.0	0.5	-1.0	2.0	-1.0	2.0	-1.0	2.0	-1.0	2.0
-1.0	0.5	-1.0	0.5	-1.0	0.5	-1.0	0.5	-1.0	2.0	-1.0	2.0	-1.0	2.0	-1.0	2.0

Table 2 probability of mistake p=.01

0.0	0.0	1.5	1.5	1.0	1.5	3.0	3.0	0.0	0.0	1.5	1.5	1.5	2.0	3.0	3.0
0.0	0.5	0.7	1.0	0.0	0.0	1.7	1.7	0.0	0.7	0.7	1.0	1.2	2.0	2.5	2.5
-0.5	0.7	1.0	1.0	0.2	2.9	1.7	2.0	-0.5	0.7	1.7	1.7	1.2	3.0	3.0	3.0
-0.5	1.0	1.0	1.0	-0.5	1.0	1.0	1.0	-0.5	1.0	1.0	1.0	1.0	2.5	2.5	2.5
-0.3	0.0	1.0	1.5	0.5	1.0	2.9	3.0	-0.3	0.0	1.0	1.5	1.3	2.0	3.0	3.0
-0.5	0.0	-0.9	1.0	-0.3	0.0	1.0	1.7	-0.6	1.0	-0.9	1.3	1.0	2.0	1.8	2.5
-1.0	0.3	0.3	1.0	-1.0	1.0	1.0	1.7	-1.0	1.0	1.0	1.7	1.0	3.0	3.0	3.0
-1.0	0.3	0.0	1.0	-1.0	0.3	0.3	1.0	-1.0	1.3	-0.9	1.3	0.8	2.5	1.8	2.5
0.0	0.0	1.5	1.5	1.0	2.0	2.9	3.0	0.0	0.0	1.6	1.6	1.3	2.0	2.6	2.7
0.0	0.7	0.7	1.0	0.0	1.0	1.0	1.3	0.0	1.0	1.0	1.3	1.0	2.0	2.0	2.0
-0.5	0.7	0.3	1.0	0.2	2.9	1.0	2.9	0.0	1.0	2.0	2.0	1.0	2.6	2.3	2.5
-0.5	1.0	0.3	1.0	-0.5	1.3	0.3	1.3	0.0	1.3	2.0	1.5	0.8	2.0	2.0	2.0
-0.5	0.3	0.3	1.0	0.0	1.0	1.0	1.7	0.0	1.0	1.0	1.7	1.0	2.0	2.0	2.5
-0.7	0.4	-1.0	0.5	-0.6	0.4	-0.9	0.5	0.0	2.0	0.0	2.0	0.7	2.0	1.0	2.0
-1.0	0.5	-1.0	0.5	-1.0	1.0	-0.9	1.0	0.0	2.0	1.0	2.0	0.7	2.3	1.5	2.3
-1.0	0.5	-1.0	0.5	-1.0	0.5	-1.0	0.5	0.0	2.0	0.5	2.0	0.5	2.0	1.0	2.0

Table 3 probability of mistake p=.05

0.1	0.1	1.5	1.5	1.0	1.5	2.8	2.8	0.1	0.1	1.5	1.5	1.5	2.0	2.9	2.9
0.0	0.5	0.7	1.0	0.1	0.2	1.6	1.6	0.0	0.7	0.7	1.0	1.2	1.9	2.4	2.4
-0.4	0.7	1.0	1.0	0.3	2.7	1.6	1.9	-0.4	0.7	1.6	1.6	1.2	2.8	2.9	2.9
-0.4	0.9	0.9	1.0	-0.4	1.0	1.0	1.1	-0.4	1.0	1.0	1.1	1.0	2.4	2.4	2.4
-0.3	0.1	1.0	1.5	0.5	1.0	2.7	2.8	-0.2	0.2	1.0	1.5	1.3	2.0	2.9	2.9
-0.4	0.1	-0.7	1.0	-0.2	0.1	1.0	1.6	-0.5	1.0	-0.7	1.3	1.0	1.9	1.8	2.4
-0.9	0.4	0.4	1.0	-0.8	1.0	1.0	1.6	-0.8	1.0	1.0	1.6	1.0	2.8	2.8	2.9
-0.9	0.4	0.1	1.0	-0.9	0.4	0.4	1.0	-0.8	1.3	-0.7	1.3	0.8	2.4	1.7	2.4
0.0	0.1	1.4	1.5	1.0	1.9	2.7	2.8	0.1	0.2	1.6	1.6	1.3	2.0	2.6	2.6
0.0	0.7	0.7	1.0	0.1	1.0	1.0	1.3	0.1	1.0	1.0	1.3	1.0	1.9	1.9	2.0
-0.4	0.7	0.4	1.0	0.2	2.7	1.0	2.7	0.1	1.0	1.9	1.9	1.0	2.5	2.2	2.4
-0.4	1.0	0.4	1.0	-0.4	1.3	0.4	1.3	0.1	1.3	1.8	1.5	0.8	2.0	1.9	2.0
-0.4	0.3	0.3	1.0	0.1	1.0	1.0	1.7	0.1	1.0	1.0	1.7	1.0	1.9	1.9	2.4
-0.6	0.4	-0.8	0.5	-0.6	0.4	-0.7	0.6	0.1	1.8	0.1	1.9	0.7	1.9	1.0	2.0
-0.9	0.5	-0.8	0.5	-0.9	1.0	-0.7	1.0	0.0	1.8	1.0	1.9	0.7	2.2	1.5	2.3
-0.9	0.5	-0.8	0.5	-0.9	0.5	-0.8	0.5	0.0	1.9	0.5	1.9	0.5	1.9	1.0	2.0

Table 4 probability of mistake p=.1

0.1	0.2	1.4	1.4	1.0	1.5	2.7	2.7	0.1	0.2	1.5	1.5	1.5	1.9	2.8	2.8
0.1	0.5	0.7	1.0	0.2	0.4	1.6	1.6	0.1	0.7	0.7	1.1	1.2	1.9	2.3	2.4
-0.3	0.7	1.0	1.0	0.3	2.4	1.6	1.9	-0.3	0.7	1.6	1.6	1.2	2.6	2.7	2.8
-0.3	0.9	0.9	1.0	-0.3	1.0	1.0	1.1	-0.3	1.0	1.0	1.1	1.0	2.3	2.3	2.3
-0.2	0.2	1.0	1.4	0.6	1.0	2.5	2.7	-0.1	0.3	1.0	1.5	1.3	1.9	2.7	2.8
-0.3	0.2	-0.4	1.0	-0.1	0.3	1.0	1.6	-0.4	1.0	-0.5	1.3	1.0	1.9	1.7	2.4
-0.8	0.4	0.4	1.0	-0.6	1.0	1.0	1.6	-0.6	1.0	1.0	1.6	1.0	2.6	2.6	2.8
-0.8	0.4	0.1	1.0	-0.7	0.4	0.4	1.0	-0.6	1.3	-0.4	1.3	0.8	2.3	1.7	2.3
0.1	0.2	1.4	1.4	1.0	1.8	2.5	2.6	0.1	0.3	1.5	1.6	1.3	1.9	2.5	2.5
0.1	0.7	0.7	1.0	0.1	1.0	1.0	1.3	0.1	1.0	1.0	1.3	1.0	1.9	1.9	1.9
-0.3	0.7	0.4	1.0	0.3	2.5	1.0	2.4	0.1	1.0	1.7	1.8	1.0	2.4	2.1	2.3
-0.4	0.9	0.4	1.0	-0.3	1.3	0.4	1.3	0.1	1.3	1.6	1.5	0.8	1.9	1.8	1.9
-0.4	0.3	0.3	1.0	0.1	1.0	1.0	1.7	0.1	1.0	1.0	1.7	1.0	1.9	1.9	2.3
-0.5	0.4	-0.6	0.6	-0.5	0.5	-0.5	0.6	0.1	1.7	0.2	1.8	0.7	1.9	1.0	1.9
-0.8	0.5	-0.7	0.6	-0.7	1.0	-0.5	1.0	0.1	1.7	1.0	1.8	0.7	2.1	1.5	2.2
-0.8	0.5	-0.7	0.6	-0.8	0.6	-0.7	0.6	0.1	1.8	0.6	1.8	0.6	1.9	1.0	1.9

Table 5 probability of mistake p=.2

0.2	0.4	1.3	1.4	1.0	1.4	2.4	2.4	0.2	0.4	1.4	1.5	1.4	1.8	2.6	2.6
0.1	0.6	0.8	1.1	0.3	0.6	1.5	1.6	0.2	0.8	0.8	1.2	1.2	1.8	2.1	2.2
-0.2	0.7	0.9	1.1	0.4	1.9	1.5	1.8	-0.0	0.8	1.4	1.5	1.2	2.3	2.5	2.5
-0.2	0.8	0.8	1.0	-0.1	1.0	1.0	1.2	-0.1	1.0	1.0	1.2	1.0	2.1	2.1	2.2
-0.1	0.3	0.9	1.4	0.6	1.1	2.1	2.4	0.0	0.5	1.1	1.5	1.3	1.8	2.4	2.6
-0.2	0.4	-0.1	1.0	0.0	0.5	1.0	1.5	-0.1	1.0	-0.0	1.3	1.0	1.8	1.7	2.2
-0.5	0.5	0.5	1.0	-0.3	1.0	1.0	1.5	-0.3	1.0	1.0	1.5	1.0	2.3	2.3	2.5
-0.5	0.5	0.2	0.9	-0.5	0.6	0.5	1.1	-0.3	1.2	0.1	1.3	0.8	2.0	1.6	2.2
0.2	0.4	1.2	1.4	0.9	1.6	2.1	2.3	0.3	0.5	1.4	1.5	1.3	1.8	2.3	2.3
0.1	0.7	0.7	1.0	0.3	1.0	1.0	1.3	0.3	1.0	1.0	1.3	1.0	1.7	1.7	1.9
-0.2	0.7	0.5	1.0	0.3	2.0	1.0	2.1	0.2	1.0	1.5	1.6	1.0	2.1	2.0	2.2
-0.2	0.8	0.4	0.9	-0.1	1.2	0.5	1.2	0.2	1.2	1.4	1.4	0.8	1.8	1.7	1.9
-0.2	0.4	0.4	1.0	0.2	1.0	1.0	1.6	0.2	1.0	1.0	1.6	1.0	1.8	1.8	2.2
-0.3	0.5	-0.3	0.6	-0.3	0.6	-0.1	0.8	0.2	1.5	0.4	1.6	0.7	1.7	1.1	1.8
-0.6	0.5	-0.4	0.6	-0.4	0.9	-0.1	1.1	0.2	1.5	0.9	1.7	0.7	2.0	1.4	2.1
-0.6	0.5	-0.4	0.6	-0.6	0.6	-0.4	0.7	0.2	1.6	0.6	1.6	0.6	1.8	1.0	1.8

Table 6 probability of mistake p=.4

0.4	0.7	1.2	1.3	1.0	1.3	1.8	1.9	0.5	0.8	1.3	1.4	1.3	1.6	2.1	2.2
0.3	0.7	0.8	1.1	0.6	0.9	1.3	1.5	0.4	0.9	1.0	1.2	1.1	1.6	1.8	1.9
0.1	0.7	0.9	1.1	0.6	1.4	1.3	1.6	0.3	0.9	1.3	1.4	1.1	1.8	2.0	2.1
0.1	0.7	0.7	1.0	0.3	1.0	1.0	1.3	0.3	1.0	1.0	1.3	1.0	1.7	1.7	1.9
0.2	0.6	0.9	1.2	0.7	1.1	1.6	1.8	0.4	0.8	1.1	1.4	1.2	1.6	2.0	2.2
0.1	0.6	0.4	1.0	0.4	0.8	1.0	1.4	0.3	1.0	0.6	1.3	1.0	1.6	1.5	1.9
-0.1	0.6	0.6	1.0	0.2	1.0	1.0	1.4	0.2	1.0	1.0	1.4	1.0	1.8	1.8	2.1
-0.1	0.6	0.4	0.9	0.0	0.7	0.7	1.1	0.2	1.1	0.6	1.3	0.9	1.7	1.4	1.9
0.4	0.7	1.1	1.2	0.9	1.4	1.6	1.8	0.5	0.8	1.3	1.4	1.2	1.6	1.9	2.0
0.3	0.7	0.7	1.0	0.5	1.0	1.0	1.3	0.5	1.0	1.0	1.3	1.0	1.5	1.5	1.7
0.1	0.7	0.6	1.0	0.5	1.4	1.0	1.6	0.4	1.0	1.2	1.4	1.0	1.7	1.6	1.9
0.1	0.8	0.5	0.9	0.2	1.0	0.7	1.2	0.4	1.1	1.1	1.3	0.9	1.6	1.4	1.7
0.1	0.6	0.6	1.0	0.4	1.0	1.0	1.4	0.4	1.0	1.0	1.4	1.0	1.6	1.6	1.9
-0.0	0.6	0.2	0.8	0.1	0.7	0.4	0.9	0.4	1.2	0.6	1.3	0.8	1.5	1.1	1.6
-0.2	0.6	0.2	0.8	0.0	0.9	0.4	1.1	0.4	1.2	0.9	1.4	0.8	1.6	1.3	1.8
-0.2	0.6	0.1	0.7	-0.1	0.7	0.2	0.8	0.4	1.2	0.7	1.3	0.7	1.5	1.0	1.6

Table 7 probability of mistake p=0

X_0	h=0
E	$x_9.=59$ $x_{10}=.19$ $x_{11}=.12$ $x_{12}=.08$ $x_i=.01$ i=13..16
V1	$x_9=.42$ $x_{10}=.58$
V2	$x_9.=10$ $x_{10}=.36$ $x_{11}=.21$ $x_{12}=.33$
V3	$x_9=.97$ $x_{10}=.03$
V4	$x_9=.85$ $x_{10}=.09$ $x_{11}=.03$ $x_{12}=.03$
V5	$x_9=.72$ $x_{10}=.28$
V6	$x_9=.74$ $x_{10}=.08$ $x_{11}=.14$ $x_{12}=.03$
V7	$x_9=.93$ $x_{10}=.05$ $x_{11}=.01$ $x_{12}=.01$
V8	$x_9=.77$ $x_{10}=.07$ $x_{11}=.14$ $x_{12}=.02$
V9	$x_9=.92$ $x_i=.01$ i=10..16
V10	$x_9=.03$ $x_{10}=.93$ $x_{11}=.02$ $x_{12}=.01$
V11	$x_9=.02$ $x_{10}=.01$ $x_{11}=.95$ $x_{12}=.01$
V12	$x_9=.03$ $x_{10}=.02$ $x_{11}=.01$ $x_{12}=.94$
V13	$x_9=.75$ $x_{10}=.24$ $x_{11}=.01$ $x_{12}=.01$
V14	$x_9=.75$ $x_{10}=.24$ $x_{11}=.01$ $x_{12}=.01$
V15	$x_9=.75$ $x_{10}=.24$ $x_{11}=.01$ $x_{12}=.01$
V16	$x_9=.75$ $x_{10}=.24$ $x_{11}=.01$ $x_{12}=.01$
V17	$x_9=.48$ $x_{10}=.52$
V18	$x_9=.12$ $x_{10}=.35$ $x_{11}=.21$ $x_{12}=.32$
V19	$x_9=.98$ $x_{10}=.02$
V20	$x_9=.85$ $x_{10}=.09$ $x_{11}=.03$ $x_{12}=.03$
V21	$x_9=.48$ $x_{10}=.52$
V22	$x_9=.62$ $x_{10}=.06$ $x_{11}=.26$ $x_{12}=.05$
V23	$x_9=.95$ $x_{10}=.04$
V24	$x_9=.72$ $x_{10}=.07$ $x_{11}=.18$ $x_{12}=.02$
V25	$x_9=.48$ $x_{10}=.52$
V26	$x_9=.57$ $x_{10}=.27$ $x_{11}=.07$ $x_{12}=.08$
V27	$x_{11}=.02$ $x_{27}=.95$ $x_i=.01$ i=12, 28, 31
V28	$x_{28}=.93$ $x_{27}=.03$ $x_i=.02$ i=11, 31
V29	$x_9=.48$ $x_{10}=.52$
V30	$x_9=.70$ $x_{10}=.08$ $x_{11}=.15$ $x_{12}=.05$
V31	$x_9=.88$ $x_{10}=.07$ $x_{11}=.03$ $x_{12}=.02$
V32	$x_9=.79$ $x_{10}=.08$ $x_{11}=.11$ $x_{12}=.02$

Table 8 probability of mistake p=.01

x_0	h=.01	h=0
E		$x_{11}=1$
V1	$x_{11}=.96$	$x_{11}=1$
V2	$x_{11}=.96$	$x_{11}=1$
V3	$x_{11}=.96$	$x_9=1$
V4	$x_{11}=.96$	$x_{11}=1$
V5	$x_{11}=.96$	$x_9=1$
V6	$x_{11}=.96$	$x_{11}=1$
V7	$x_{11}=.96$	$x_{11}=1$
V8	$x_{11}=.96$	$x_{11}=1$
V9	$x_{11}=.96$	$x_{11}=1$
V10	$x_{11}=.96$	$x_{11}=1$
V11	$x_{11}=.96$	$x_{11}=1$
V12	$x_{11}=.96$	$x_{11}=1$
V13	$x_{11}=.96$	$x_{11}=1$
V14	$x_{11}=.96$	$x_{11}=1$
V15	$x_{11}=.96$	$x_{11}=1$
V16	$x_{11}=.96$	$x_{11}=1$

Table 9 probability of mistakes p=.05

x_0	h=.01	h=0
E	x_{11}=.98	x_{11}=1
V1	x_{11}=.98	x_1=1
V2	x_{11}=.98	x_{11}=1
V3	x_{11}=.98	x_9=1
V4	x_{11}=.98	x_{11}=1
V5	x_{11}=.98	x_9=1
V6	x_{11}=.98	x_{11}=1
V7	x_{11}=.98	x_{11}=1
V8	x_{11}=.98	x_{11}=1
V9	x_{11}=.98	x_{11}=1
V10	x_{11}=.98	x_9=1
V11	x_{11}=.98	x_{11}=1
V12	x_{11}=.98	x_{11}=1
V13	x_{11}=.98	x_{11}=1
V14	x_{11}=.98	x_{11}=1
V15	x_{11}=.98	x_{11}=1
V16	x_{11}=.98	x_{11}=1

Table 10 probability of mistakes p=.1

x_0	h=.01	h=0
E	x_{11}=.98	x_{11}=1
V1	x_{11}=.98	x_1=1
V2	x_{11}=.98	x_{11}=1
V3	x_{11}=.98	x_9=1
V4	x_{11}=.98	x_9=1
V5	x_{11}=.98	x_9=1
V6	x_9=.85 x_{11}=.04 *	x_9=1
V7	x_{11}=.98	x_9=1
V8	x_{11}=.98	x_9=1
V9	x_9=.85 x_{11}=.04 *	x_9=1
V10	x_{11}=.98	x_1=1
V11	x_{11}=.98	x_{11}=1
V12	x_{11}=.98	x_{11}=1
V13	x_{11}=.98	x_9=1
V14	x_{11}=.98	x_9=1
V15	x_{11}=.98	x_9=1
V16	x_{11}=.98	x_9=1

* With h=.001 the steady state is x_{11}=1.

Table 11 probability of mistakes p=.2

x_0	h=.01	h=0
E	x_9=.88 x_{10}=.03 x_i=.01 i=11..16 x_1=.01	x_{11}=1
V1	x_9=.88 x_{10}=.03 x_i=.01 i=11..16 x_1=.01	x_1=1
V2	x_9=.88 x_{10}=.03 x_i=.01 i=11..16 x_1=.01	x_9=1
V3	x_9=.88 x_{10}=.03 x_i=.01 i=11..16 x_1=.01	x_9=1
V4	x_9=.88 x_{10}=.03 x_i=.01 i=11..16 x_1=.01	x_9=1
V5	x_9=.88 x_{10}=.03 x_i=.01 i=11..16 x_1=.01	x_9=1
V6	x_9=.88 x_{10}=.03 x_i=.01 i=11..16 x_1=.01	x_9=1
V7	x_9=.88 x_{10}=.03 x_i=.01 i=11..16 x_1=.01	x_9=1
V8	x_9=.88 x_{10}=.03 x_i=.01 i=11..16 x_1=.01	x_9=1
V9	x_9=.88 x_{10}=.03 x_i=.01 i=11..16 x_1=.01	x_9=1
V10	x_9=.88 x_{10}=.03 x_i=.01 i=11..16 x_1=.01	x_1=1
V11	x_{11}=.96	x_{11}=1
V12	x_9=.88 x_{10}=.03 x_i=.01 i=11..16 x_1=.01	x_9=1
V13	x_9=.88 x_{10}=.03 x_i=.01 i=11..16 x_1=.01	x_9=1
V14	x_9=.88 x_{10}=.03 x_i=.01 i=11..16 x_1=.01	x_9=1
V15	x_9=.88 x_{10}=.03 x_i=.01 i=11..16 x_1=.01	x_9=1
V16	x_9=.88 x_{10}=.03 x_i=.01 i=11..16 x_1=.01	x_9=1

Table 12 probability of mistakes p=.4

x_0	h=.01	h=0
E	x_1=.94	x_1=1
V1	x_1=.94	x_1=1
V2	x_1=.94	x_1=1
V3	x_1=.94	x_1=1
V4	x_1=.94	x_1=1
V5	x_1=.94	x_1=1
V6	x_9=.90	x_9=1
V7	x_1=.94	x_1=1
V8	x_1=.94	x_1=1
V9	x_9=.90	x_9=1
V10	x_9=.90	x_1=1
V11	x_1=.94	x_1=1
V12	x_1=.94	x_1=1
V13	x_1=.94	x_1=1
V14	x_1=.94	x_1=1
V15	x_1=.94	x_1=1
V16	x_1=.94	x_1=1

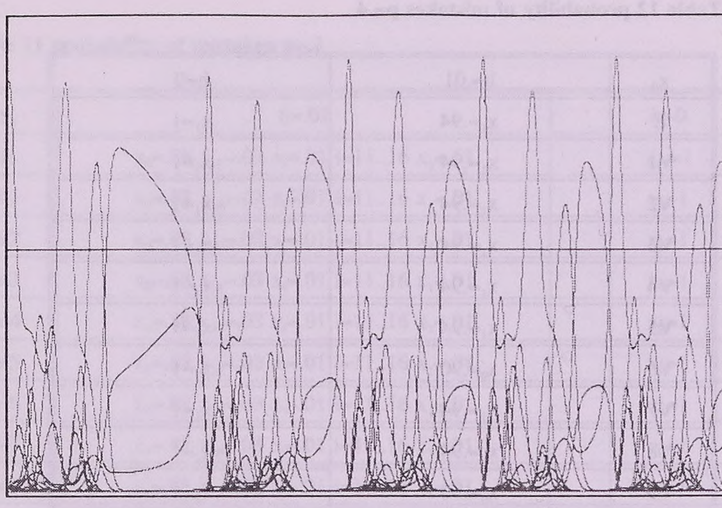

Figure 1, V10 h=.001 p=.1 T=6000

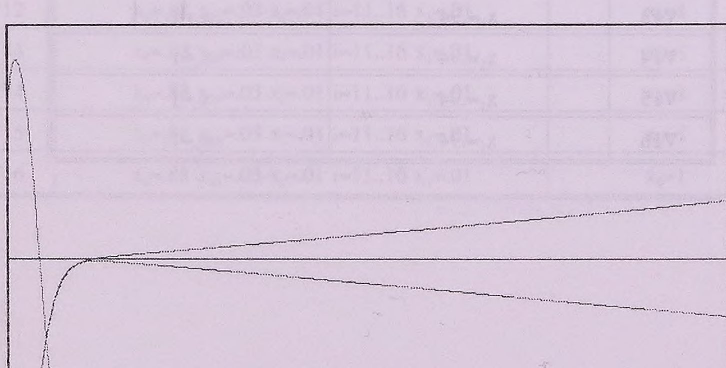

Figure 2, V5 h=0 p=.01 T=2000

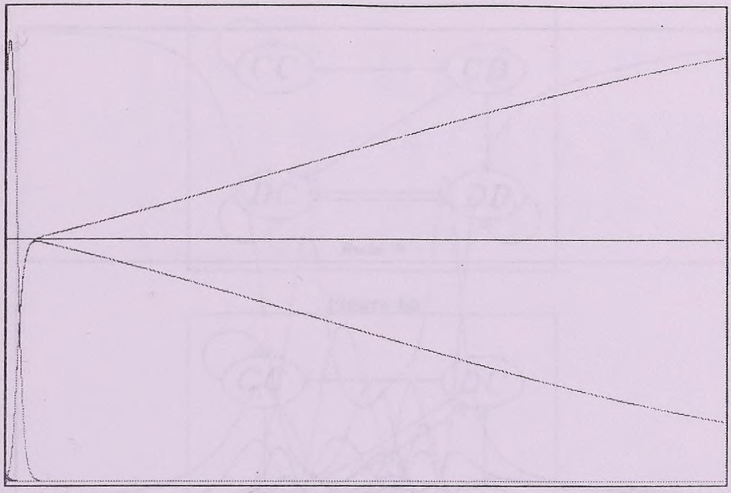

Figure 2b, V5 h=0 p=.01 T=6000

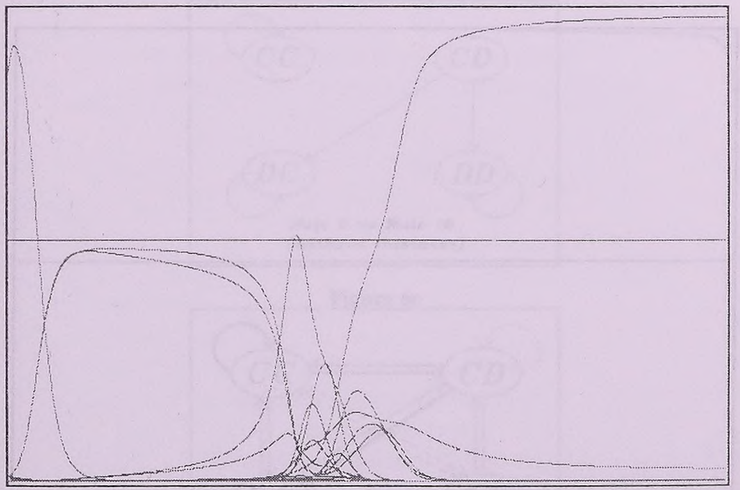

Figure 3, V5 h=.01 p=.01 T=2000

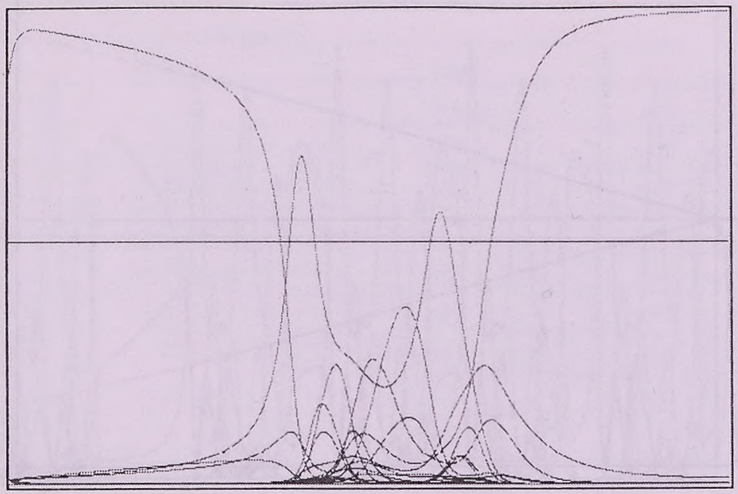

Figure 4, V1 h=.01 p=.05 T=2000

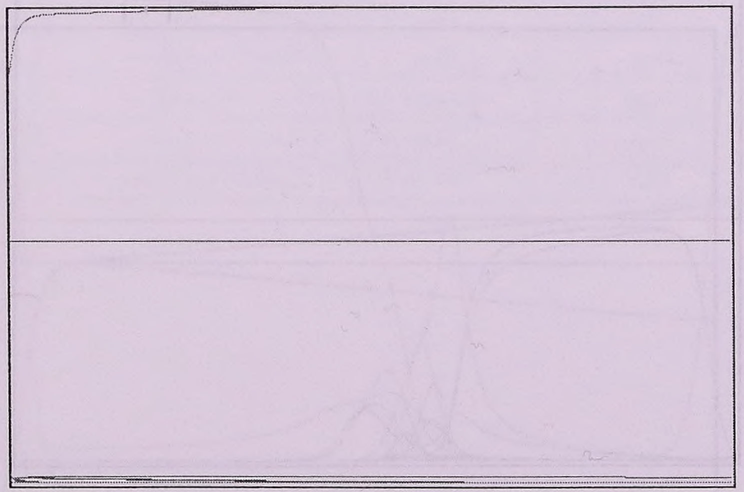

Figure 5 V1 h=0 p=.05 T=2000

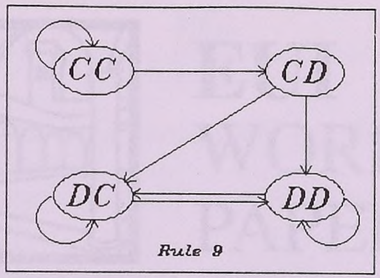

Figure 6a

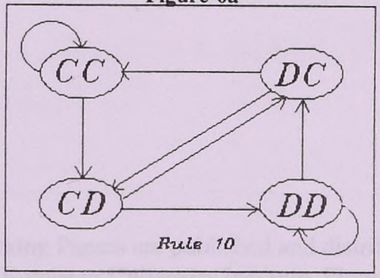

Figure 6b

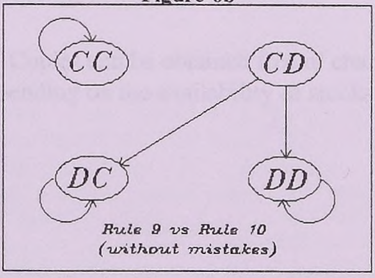

Figure 6c

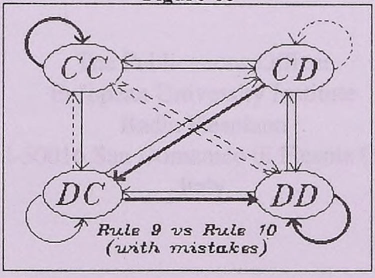

Figure 6d

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